Week 2 - Monday

COMP 1800

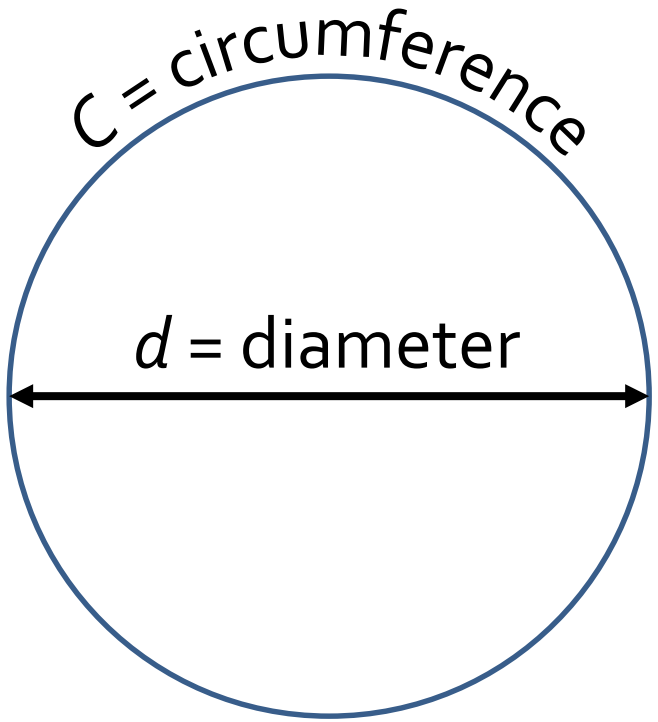
Last time

- What did we talk about last time?
- Functions
- For loops
- Work time for Assignment 1

Questions?

What is π?

- The ratio of the circumference to the diameter of any circle
  - $\pi = \frac{C}{d}$
- Ancient Greek mathematician Archimedes tried to calculate its value in the third century BCE
- Even though we have calculated trillions of digits of π, there are always more, and it's hard to figure them out



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5982534904 2875546873 1159562863 8823537875 9375195778 1857780532
1712268066 1300192787 6611195909 2164201989

Attempts to approximate π

- The number π is irrational, meaning that it's neither a terminating decimal nor one that repeats with a simple pattern
- In base 10, we can only ever write an approximation of π
- In ancient times, Archimedes knew it was around 3.14
- A Chinese mathematician named Zu Chongzhi had it correct to seven digits after the decimal point in 480 CE
- In the early 16th century, it was known to 100 digits
- With electronic computers, the number of digits known in the 20th century went from 2,037 in 1949 to over 206 billion in 1999
- The 2022 record holder is exactly 100 trillion digits

math module

- In order to do some of our π calculations, we're going to need more than just basic arithmetic
- The math module has standard functions like square root and trigonometry
- To use it, you have to import it:

```
import math
```

Some `math` functions

Return type	Name	Job
Integer	`ceil( x )`	Find the ceiling of x
Integer	`floor( x )`	Find the floor of x
Floating-point	`fabs( x )`	Find the absolute value of x
Floating-point	`sin( theta )`	Find the sine of angle theta
Floating-point	`cos( theta )`	Find the cosine of angle theta
Floating-point	`tan( theta )`	Find the tangent of angle theta
Floating-point	`exp( a )`	Raise *e* to the power of a (e^a)
Floating-point	`log( a )`	Find the natural log of a
Floating-point	`pow( a, b )`	Raise a to the power of b (a^b)
Floating-point	`sqrt( a )`	Find the square root of a
Floating-point	`degrees( radians )`	Convert radians to degrees
Floating-point	`radians( degrees )`	Convert degrees to radians

Using `math` functions

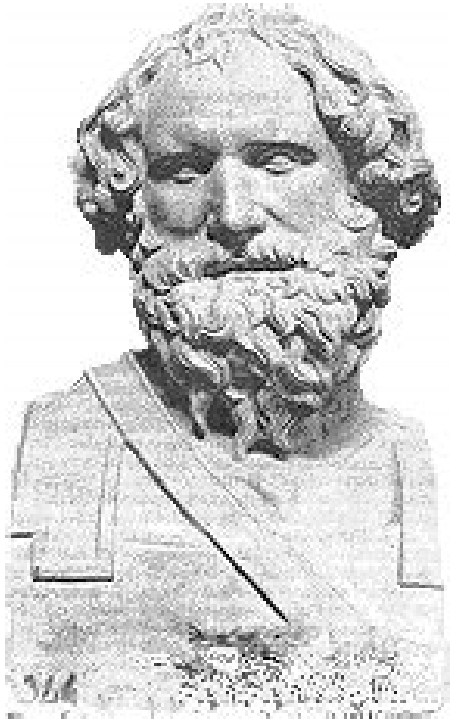
- After importing **`math`**, you still say **`math.`** before the name of a function
- For example, to compute the cosine of 2.6 radians, you can do the following:

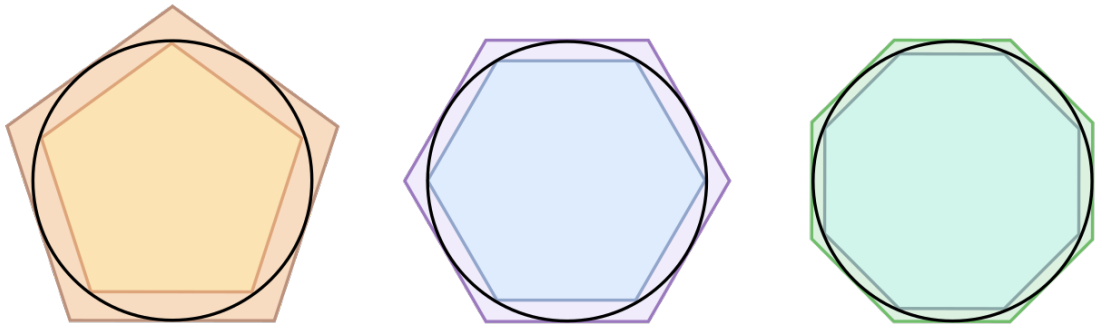
```
import math
result = math.cos(2.6)
```

- Note that all the trigonometry functions take radians, not degrees

Archimedes' method

- Start with a unit circle
- Draw polygons inside the circle
- Calculate their perimeters
- As the number of sides gets larger, the perimeters more closely approximate the circumference of the circle

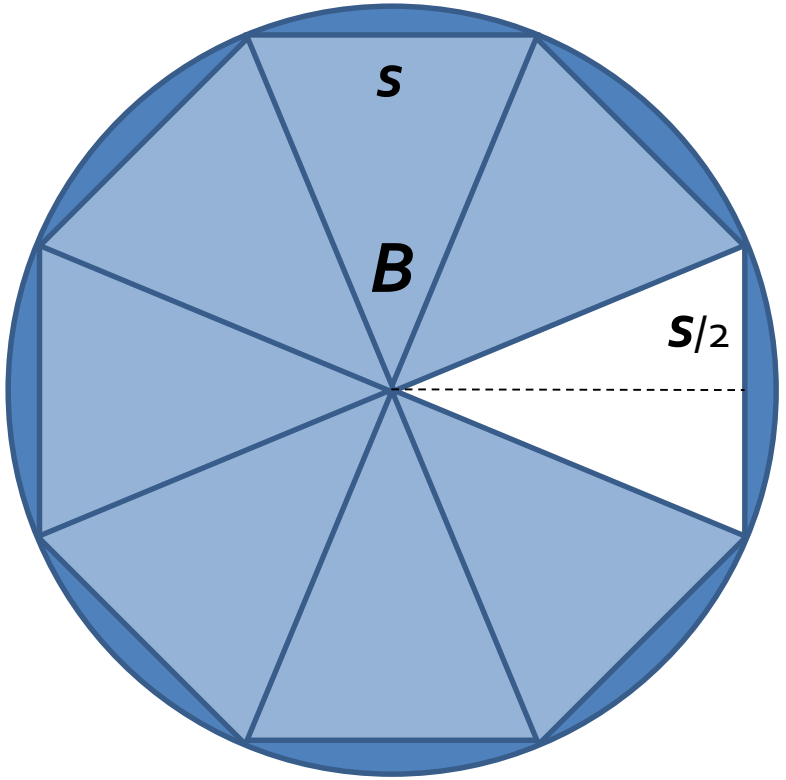




Math behind Archimedes

- An ***n***-sided polygon inside of a unit circle is made up of ***n*** triangles, fit together like a pizza
- Angle ***B*** is 360 / ***n***
- Side ***s*** is opposite ***B***
- Each triangle is made up of two right triangles with a height of ½***s***
- Because it's a unit circle, ***r*** = 1, so $\frac{s}{2} = \sin(\frac{B}{2})$
- The total perimeter is:

$$2n\frac{s}{2} = n \cdot \sin(\frac{B}{2})$$



Writing a function

- We can write a function that will take different values for sides, the number of sides that the polygon will have
- Each one will give us different approximations to π
- Let's complete the following function definition, using the math on the previous slide:

```
def archimedes(sides):
```

`return` statements

- Like most code in Python, the code inside of a function executes line by line
- Of course, you are allowed to put loops inside functions
- You can also put in **`return`** statements
- A function will stop executing and jump back to wherever it was called from when it hits a **`return`**
- The **`return`** statement is where you put the value that will be given back to the caller

The `range()` function

- The **`range()`** function produces a sequence of values that a variable will take on
- With only a single parameter **`n`**, the sequences of numbers is 0, 1, 2,…,**`n`** – 1 (but not **`n`**)

```
for i in range(100):
```

- With two parameters, **`a`** and **`b`**, the sequence starts at **`a`** and goes up to **`b`** – 1 (but not **`b`**)

```
for i in range(10,20):
```

- With three parameters, **`a`**, **`b`**, and **`step`**, the sequence starts at **`a`** and goes almost up to (but not including) **`b`**, taking steps of size **`step`**

```
for i in range(10,20,5):
```

Various approximation values

- Since we have a function, we can put it inside a **`for`** loop to see how the approximation gets better with larger numbers of sides
- We can use the third kind of **`range`** statement, starting at 8, going up to (but not including) 100, and jumping in steps of 8 as well

```
for sides in range(8, 100, 8):
  print (sides, archimedes(sides))
```

Improved approximation

- Archimedes' real method is supposed to put a polygon inside *and* outside the circle
- The average of their perimeters should be a better approximation of π than either one alone
- Let's make an improved version of our function that finds a better approximation

```
def betterArchimedes(sides):
```

Patterns

- A **design pattern** is a problem-solving technique in coding in which there is a standard way of do something that is used a lot
- **Accumulator Pattern**
  - Produce a result by iterating over a sequence of values and accumulate their sum (or other aggregation) along the way
- This example finds the sum of numbers from 1 up to 10:

```
>>> acc = 0
>>> for x in range(1, 11):
        acc = acc + x
>>> acc
    55
```

Gottfried Wilhelm von Leibniz

- Leibniz invented calculus at the same time as Newton
- He created an approximation for π:
  - $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$
  - $\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \cdots$
- How can we do this in Python?
- We need an Accumulator Pattern
- We also need a variable for the denominator that increases by 2 every time so that it's always odd
- We also need a way to subtract every other term

John Wallis

- John Wallis was a 17th century British mathematician who is believed to have come up with the ∞ symbol for infinity
- Of course, he also found an approximation for π
  - $\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \ldots$

Wallis approximation

- $\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdots$

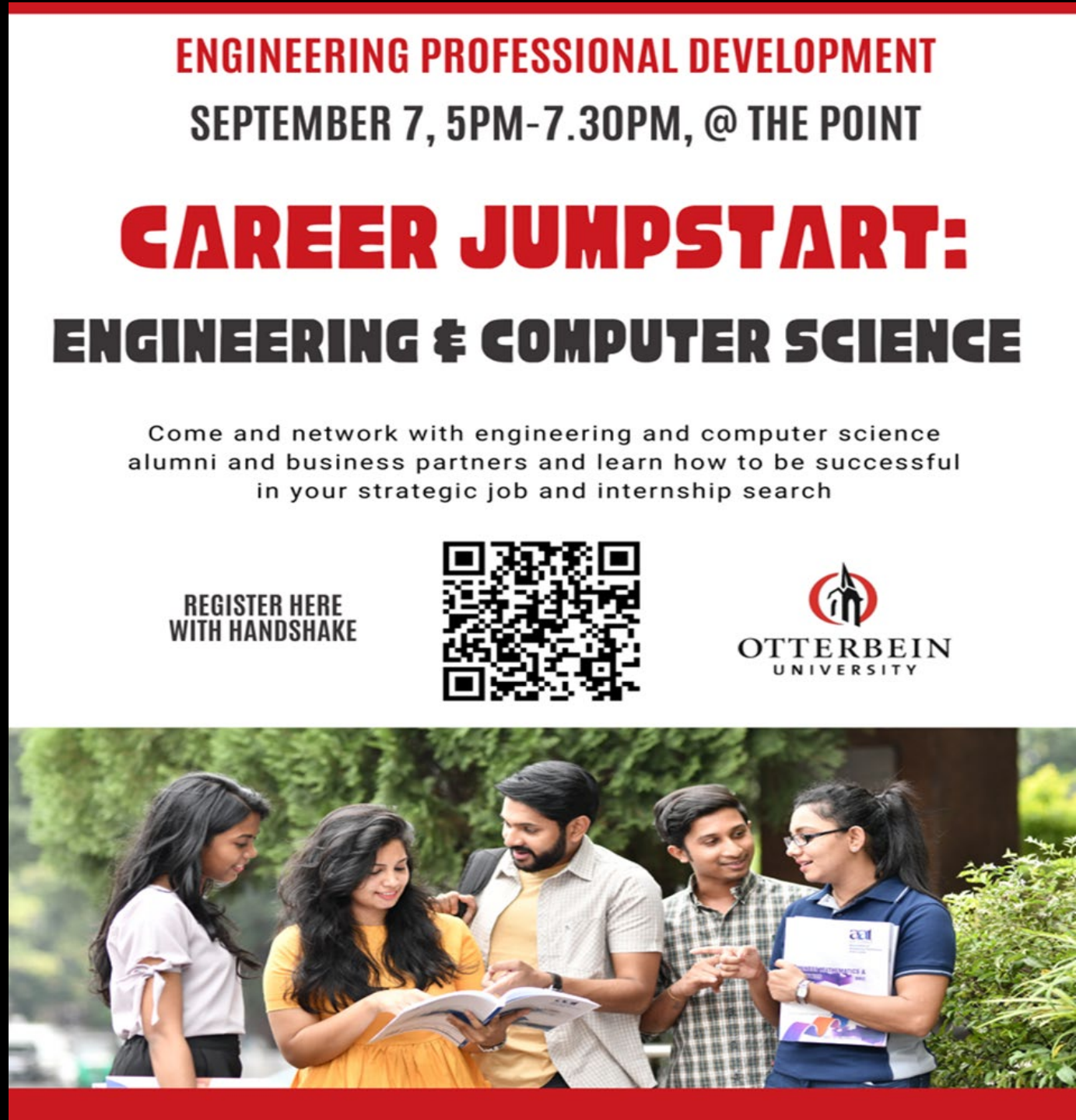
- As with Leibniz, we will use the Accumulator Pattern
- Although the Accumulator Pattern often adds things up, we'll be multiplying stuff as we go
  - When summing, we start with 0
  - When multiplying, we start with 1
- Note that both the numerator and the denominator are used twice in a row before increasing by two
  - But they change on opposite turns!

Upcoming

Next time…

- Simulation and random numbers
- We'll do a Monte Carlo approximation of π
- We'll also talk about Boolean variables and selection statements

- 20 employers in the fields of Engineering and Computer Science
- 20 alumni members attending
- Free professional LinkedIn headshots
- Plenty of food and great conversations
- Build new connections on LinkedIn
- Door prizes
- Network with people in your field
- Learn about possible internships
- Gain new insights about your major
- Required event for all sophomores



Reminders

- Keep reading Chapter 2 of the textbook
- Emergency elections for CS Club
  - Do you want to have a voice in CS Club?
  - Come vote at 4 p.m. this Wednesday, August 30 in Point 113!